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Tugas Metode Charpit

11. Dapatkan penyelesaian PD berikut ini.

a). $p^2 + q^2 = 10pq$

Penyelesaian :

① Bentuk PDP sebagai fungsi implisit $f(x, y, u, p, q) = 0$

$\Rightarrow p^2 + q^2 - 10pq = 0$

② Diferensiasikan f secara parsial terhadap x, y, u, p, dan q

$$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0 \quad \frac{\partial f}{\partial u} = 0 \quad \frac{\partial f}{\partial p} = 2p - 10q \quad \frac{\partial f}{\partial q} = 2q - 10p$$

③ Dapatkan persamaan Charpit yang jadi bentuk PDB

$$\frac{dx}{-\partial f/\partial p} = \frac{dy}{-\partial f/\partial q} = \frac{du}{-\left(p\,\partial f/\partial p + q\,\partial f/\partial q\right)} = \frac{dp}{\left(\partial f/\partial x + p\frac{\partial f}{\partial z}\right)} = \frac{dq}{\left(\partial f/\partial y + q\frac{\partial f}{\partial z}\right)} = \frac{df}{0}$$

Diperoleh :

$$\frac{dx}{10q - 2p} = \frac{dy}{10p - 2q} = \frac{du}{-\left(2p^2 - 10pq + 2q^2 - 10pq\right)} = \frac{dp}{\left(0 + p(0)\right)} = \frac{dq}{\left(0 + q(0)\right)} = \frac{df}{0}$$

$$\Rightarrow \frac{dp}{0} = \frac{dq}{0} = \frac{du}{-\left(2(p^2+q^2) - 20pq\right)} = \frac{dx}{10q - 2p} = \frac{dy}{10p - 2q}$$

④ Selesaikan pers. Charpit untuk dapat p maupun q dengan manfaatkan juga PD awal.

$$\frac{dp}{0} = \frac{dq}{0} \rightarrow p = \alpha \text{ dan } q = \beta$$

⑤ Substitusikan p dan q pada diferensial total u, yaitu $du = p\,dx + q\,dy$

$\rightarrow$ Tentukan hubungan p dan q

Anggap $p^2 + q^2 - 10pq = 0 \Longrightarrow \quad p = 5q \pm 2q\sqrt{6} \Rightarrow (5+2\sqrt{6})q$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10q \pm \sqrt{100q^2 - 4q^2}}{2}$$

$$p = \frac{10q \pm \sqrt{96q^2}}{2} = \frac{10q \pm 4q\sqrt{6}}{2} = 5q \pm 2q\sqrt{6}$$

Sehingga hasil substitusi :

$$du = p\,dx + q\,dy$$
$$= (5+2\sqrt{6})q\,dx + q\,dy$$
$$= (5+2\sqrt{6})\beta\,dx + \beta\,dy$$
$$u = (5+2\sqrt{6})\beta x + \beta y + \alpha$$

b). $p + q = pq$

▶ Fungsi Implisit :

$$p + q - pq = 0$$

▶ Diferensial

$$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0 \quad \frac{\partial f}{\partial u} = 0 \quad \frac{\partial f}{\partial p} = 1-q \quad \frac{\partial f}{\partial q} = 1-p$$

▶ Persamaan Charpit

$$\frac{dx}{-\partial f/\partial p} = \frac{dy}{-\partial f/\partial q} = \frac{du}{-\left(p\frac{\partial f}{\partial p} + q\frac{\partial f}{\partial q}\right)} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{df}{0}$$

$$\frac{dx}{-(1-q)} = \frac{dy}{-(1-p)} = \frac{du}{-(p-pq+q-pq)} = \frac{dp}{0} = \frac{dq}{0} = \frac{df}{0}$$

$$\frac{dx}{q-1} = \frac{dy}{p-1} = \frac{du}{2pq-p-q} = \frac{dp}{0} = \frac{dq}{0}$$

$\cdot>$ $\dfrac{dp}{0} = df$ $\qquad\qquad$ $\cdot>$ $\dfrac{dq}{0} = df$

$dp = 0\,df$ $\qquad\qquad$ $dq = 0\,df$

$p = \alpha$ $\qquad\qquad$ $q = \beta$

Persamaan Awal :

$$p + q = pq$$
$$p + q - pq = 0$$
$$p(1-q) + q = 0$$
$$p = \frac{q}{q-1}$$

Jadi, $p = \dfrac{\beta}{\beta - 1}$

Diperoleh :

PUPD : $du = p\,dx + q\,dy$

$$du = \left(\frac{\beta}{\beta-1}\right)dx + \beta\,dy$$
$$u = \frac{\beta}{\beta-1}x + \beta y + \alpha$$

c). $z^2(p^2+q^2+1) = 1$

- Dalam implisit

$$z^2(p^2+q^2+1) - 1 = 0$$

- Diferensial

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0 \qquad \frac{\partial f}{\partial z} = 2(p^2+q^2+1)z \qquad \frac{\partial f}{\partial p} = 2pz^2 \qquad \frac{\partial f}{\partial q} = 2qz^2$$

- Persamaan Charpit

$$\frac{dx}{-2pz^2} = \frac{dy}{-2qz^2} = \frac{dz}{-(2p^2z^2 + 2q^2z^2)} = \frac{dp}{2pz(p^2+q^2+1)} = \frac{dq}{2qz(p^2+q^2+1)} = \frac{df}{0}$$

$$\frac{dp}{2p\cancel{z(p^2+q^2+1)}} = \frac{dq}{\cancel{2}q\cancel{z(p^2+q^2+1)}}$$

$$\frac{dp}{p} = \frac{dq}{q}$$

$$\ln p = \ln q + C$$

$$e^{\ln p} = e^{\ln q + C}$$

$$p = Cq$$

Substitusi ke persamaan awal :

$$z^2(p^2+q^2+1) = 1$$

$$p^2+q^2+1 = \frac{1}{z^2}$$

$$q^2(C^2+1)+1 = \frac{1}{z^2}$$

$$q^2 = \left(\frac{1}{z^2} - 1\right)\frac{1}{C^2+1}$$

$$q = \sqrt{\frac{1}{C^2+1}\left(\frac{1}{z^2}-1\right)}$$

$$p = qC$$

$$= \sqrt{\frac{C^2}{C^2+1}\left(\frac{1}{z^2}-1\right)}$$

$$\int \frac{z}{\sqrt{1-z^2}}\, dz =$$

misal : $k = 1-z^2$

$dk = -2z\,dz \rightarrow z\,dz = \frac{dk}{-2}$

$$= \int \frac{-dk}{2\sqrt{k}}$$

$$= -\frac{1}{2} \cdot 2 \cdot \sqrt{k}$$

$$= -\sqrt{k}$$

$$= -\sqrt{1-z^2}$$

Sehingga hasil substitusi:

misal : $\sqrt{\frac{1}{c^2+1}} = s$

$$\frac{1}{c^2+1} = s^2$$

$$c^2 = \frac{1}{s^2} - 1$$

$$dz = z_x dx + z_y dy$$

$$= p\,dx + q\,dy$$

$$dz = \sqrt{\frac{c^2}{c^2+1}\left(\frac{1}{z^2}-1\right)}dx + \sqrt{\frac{1}{c^2+1}\left(\frac{1}{z^2}-1\right)}dy$$

$$\sqrt{\frac{z^2}{1-z^2}}\,dz = \sqrt{\frac{c^2}{c^2+1}}\,dx = \sqrt{\frac{1}{c^2+1}}\,dy$$

$$-\sqrt{1-z^2} = \left(\frac{1}{s^2}-1\right)s x + s y$$

$$\sqrt{1-z^2} = \left[\left(s - \frac{1}{s}\right)x + sy\right]^2$$

$$z^2 = 1 - \left[\left(s - \frac{1}{s}\right)x + sy\right]^2$$

$$= 1 - \left[\left(s - \frac{1}{s}\right)x + sy\right]^2$$

$$z = \sqrt{1 - \left[\left(s - \frac{1}{s}\right)x + sy\right]^2}$$

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12). Dapatkan PD $u = px + qy + \sqrt{1+p^2+q^2}$, penyelesaian umum...

$$u^{k} = px + qy + \sqrt{1+p^2+q^2}$$
$$u = ax + by + \sqrt{1+a^2+b^2}$$

$$\frac{\partial u}{\partial a} = x + \frac{a}{\sqrt{1+a^2+b^2}} = 0$$

$$\frac{\partial u}{\partial b} = y + \frac{b}{\sqrt{1+a^2+b^2}} = 0$$

diperoleh :
$$x = \frac{-a}{\sqrt{1+a^2+b^2}} \qquad y = \frac{b}{\sqrt{1+a^2+b^2}}$$

$$x^2 = \frac{a^2}{1+a^2+b^2} \qquad y^2 = \frac{b^2}{1+a^2+b^2}$$

Substitusi u dan z

$$u = \frac{a\left(\frac{-a}{\sqrt{1+a^2+b^2}}\right) + b\left(\frac{-b}{\sqrt{1+a^2+b^2}}\right) + \sqrt{1+a^2+b^2}}{\sqrt{1+a^2+b^2}}$$

$$u\left(\sqrt{1+a^2+b^2}\right) = -a^2 - b^2 + 1 + a^2 + b^2$$

$$u\left(\sqrt{1+a^2+b^2}\right) = -a^2 - b^2 + 1 + a^2 + b^2$$

$$u\left(\sqrt{1+a^2+b^2}\right) = 1$$

$$u = \frac{1}{\sqrt{1+a^2+b^2}}$$

$$u^2 = \frac{1}{1+a^2+b^2}$$

$$x^2 + y^2 + u^2 = \frac{a^2}{1+a^2+b^2} + \frac{b^2}{1+a^2+b^2} + \frac{1}{1+a^2+b^2}$$

$$x^2 + y^2 + z^2 = 1$$

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